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| Mathematics Design Collaborative |
| State of Georgia Department of Education |



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| **Piecewise-Defined Function** |
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| ***Characteristics of Piecewise-Functions*** |
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| INTRODUCTION TO THIS FORMATIVE ASSESSMENT LESSON | **INTRODUCTION TO THIS FORMATIVE ASSESSMENT LESSON** |

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| **MATHEMATICAL GOALS** |
| This lesson unit is intended to help you assess how well students are able to distinguish between and identify characteristics of linear, quadratic, square root, step, absolute value, and piecewise-defined functions. In particular, the lesson will help you identify and help students who have the following difficulties:   * Understand how functions can be expressed in different forms, but remain equivalent. * Identify the characteristic that distinguishes the function family to which it belongs. * Describing the characteristics of piecewise-defined functions. |

**GEORGIA STANDARDS OF EXCELLENCE**

This lesson involves mathematical content in the standards from across the grades, with emphasis on:

* **MGSE9-12.F.IF.7a** **Graph linear and quadratic functions and show intercepts, maxima, and minima (as determined by the function or by context).**
* **MGSE9-12.F.IF.7b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
* **MGSE9-12.F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
* **MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.***

This lesson also relates to the following Standards for Mathematical Practice in the CCSS:

SMP 1. Make sense of problems and persevere in solving them.

SMP 2. Reason abstractly and quantitatively.

SMP 3. Construct viable arguments and critique the reasoning of others.

SMP 6. Attend to precision.

SMP 7. Look for and make use of structure.

SMP 8. Look for and express regularity in repeated reasoning.

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| **INTRODUCTION** |
| This lesson is structured in the following way: |
| Before the Lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work, and create questions for students to answer in order to improve their solutions. |
| At the Start of the Lesson, after a whole-class interactive introduction, students work in groups on a collaborative task in which they match graphs to the algebraic representations. As they do this, they begin to link different algebraic forms of functions to particular properties of their graphs. |
| During the Lesson, students work in groups on a collaborative task in which they match graphs to the algebraic representations. As they do this, they begin to link different algebraic forms of functions to particular properties of their graphs. |
| After the Whole-Group Class Discussion, after a plenary discussion, students return to their original assessment tasks, and try to improve their own responses. |

**MATERIALS REQUIRED**

Each individual student will need:

* Two copies of the assessment tasks. (One for pre-assessment and one for post-assessment)

Each small group of students will need:

* One set of pre-cut Card Set A.
* Final Graphs Paper
* Extension group only – 5 index cards (If applicable)

The Teacher may need:

* Document Camera (Optional)

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| **TEACHER PREP REQUIRED** |
| Teacher, be advised that prior to the lesson, the following preparations/copies will need to be made: |
| * Precut *Card Set A* |

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| **TIME NEEDED:** | | | | | | | |
| For Pre-Assessment: | 20 min. | For Lesson: | 50 min. | For Post: | 20 min. |  |  |
| Special Note(s) about timing: Allow 30 min of lesson for collaborative activity. | | | | | | | |

**FRAMING FOR THE TEACHER:**

Students will be asked to match graphs of piecewise-defined functions. Students will be “tricked” by these cards, because the same functions are re-used from time to time, but with different pieces represented in the cards. The answers have been coded so as to create words. This will allow a teacher to check correctness very quickly without having to study students’ responses in the collaborative activity.

**FRAMING FOR THE STUDENTS:**

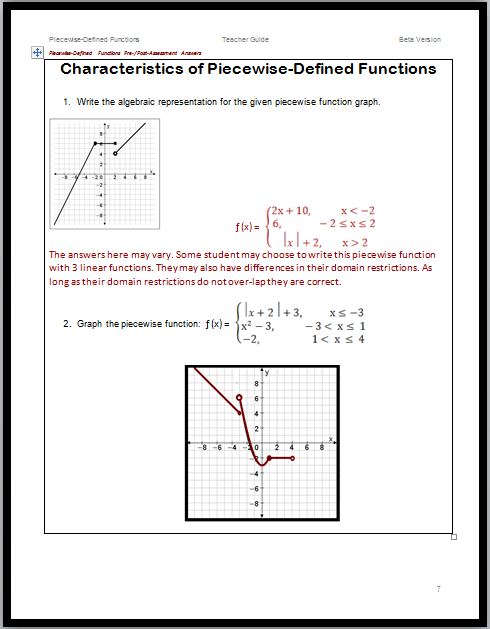
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| Say to the students: |
| *This activity will take about \_\_\_\_\_1.5\_\_\_days for us to complete.* |
| *The reason we are doing this is to be sure that you understand how to graph piecewise functions before we move on to a new idea.* |
| *You will have a chance to work with a partner to correct any misconceptions that you may have. After the partner work, you will be able to show me what you have learned!* |

PRE-ASSESSMENT BEFORE THE LESSON

**ASSESSMENT TASK:**

Characteristics of Piecewise-Defined Functions

20 Minutes



Have the students do this task in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of

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| **Piecewise-Defined Functions** |

Briefly introduce the task and help the class to understand the problem and its context.

*Read through the task and try to answer it as carefully as you can.*

*Show all your work so that I can understand your reasoning.*

It is important that students are allowed to answer the questions without your assistance, as far as possible.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to be able to answer questions such as these confidently. This is their goal.

**Assessing students’ responses**

Collect students’ responses to the task. Make some notes on what their work reveals about their current levels of understanding, and their different problem solving approaches.

We suggest that you do not score students’ work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write a list of your own questions, based on your students’ work, using the ideas that follow. You may choose to write questions on each student’s work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the end of the lesson.

Have the students do this task in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will them be able to target your help more effectively in the follow-up lesson.

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| COLLABORATION TIME/READING STUDENTS RESPONSES | |
| **You Will Not “Grade” These!** | |
| Collect students’ responses to the task. It is helpful to read students’ responses with colleagues who are also analyzing student work. Make notes (on your own paper, not on their pre-assessment) about what their work reveals about their current levels of understanding, and their approaches to the task. You will find that the misconceptions reveal themselves and often take similar paths from one student to another and even from one teacher to another. Some misconceptions seem to arise very organically in students’ thinking. Pair students in the same classes with other students who have similar misconceptions. This will help you to address the issues in fewer steps, since they’ll be together. (Note: pairs are better than larger groups for FAL’s because both must participate in order to discuss!) | |
| You will begin to construct Socrates-style questions to try and elicit understanding from students. We suggest you write a list of your own questions; however some guiding questions and prompts are also listed below as a jumping-off point. | |
| **GUIDING QUESTIONS *Here are some questions that you may find useful as you address misconceptions in class:*** | |
| ***COMMON ISSUES*** | ***SUGGESTED QUESTIONS AND PROMPTS*** |
| Student was unable to begin. | * *You are given a graph and an equation.* * *How many “pieces” does each algebraic representation need?* * *What type of equation would be used to represent each piece of the graph (linear, absolute value, quadratic, etc.)* * *What would the equation for those function families look like?* |
| Student has misunderstanding of points of discontinuity. | * *What does an open circle mean or represent in an inequality?* * *What does the closed circle represent?* |
| Student makes incorrect assumptions about domain restrictions. | * *Read the domain restrictions aloud and follow along by tracing with your finger on the graph.* |
| Student correctly answers all the questions.  (The student needs an extension to the task.) | * *Have the students match the cards just as everyone else, but then have them write a scenario that each piecewise-defined function could represent.* |
| Student was unable to begin. | * *You are given a graph and an equation.* * *How many “pieces” does each algebraic representation need?* * *What type of equation would be used to represent each piece of the graph (linear, absolute value, quadratic, etc.)* * *What would the equation for those function families look like?* |
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| LESSON DAY | |
| **SUGGESTED LESSON OUTLINE:** | |
| **Part 1: Whole-Class Introduction:** | **Time to Allot: ( 10 minutes)** |

Teacher writes student responses on the board while leading the discussion. Write the students’ names by their contributions to the discussion as you write what they have said on the board.

Ask the class to discuss the characteristics of the function families mentioned and how they are able to distinguish them from other function families. Also discuss characteristics of piecewise-defined functions.

Suggested Prompts:

* *What are the function families you covered thus far in this unit or that you remember from previous years?*
* *What are the characteristics of those function families?*
* *How are you able to distinguish one function family from other function families?*
* *What is different/significant about piecewise –defined functions from other functions that are not piecewise-defined?*
* *In what situation(s) would a piecewise-defined function be appropriate to describe a real-world situation?*
* *Explain domain restrictions – what does this mean, graphically?*

**Part 2: Collaborative Activity: Grouping Card Set A Time to Allot: (20 minutes)**

Put students into their pairs according to your analysis of student errors.

Give each small group of students a pre-cut copy of Card Set A. Explain to the students that the aim is to match the graph of each piece of the piecewise-defined function with the card that has its algebraic representation. After matching each graph to the corresponding equation, have the students draw each piece of the piece wise functions on one graph.

You may want to say:

* *Take turns matching pieces that you believe belong together.*
* *Each time you do match pieces, explain your reasoning clearly and carefully to your group.*
* *It is important that you both understand your matches. If you do not agree or understand ask your partner to explain their reasoning. You both are responsible for each other’s learning.*

The purpose of this structured group work is to encourage students to engage with each other's explanations and take collaborative responsibility for understanding.

During the Collaborative Activity, the Teacher has 3 tasks:

* Circulate to students’ whose errors you noted from the pre-assessment and support their reasoning with your guiding questions.
* Circulate to other students also to support their reason in the same way.
* Make a note of student approaches for the summary (plenary discussion). Some students have interesting and novel solutions!

**Make a note of student approaches to the task.**

Listen and watch students carefully. Note different student approaches to the task and any common mistakes. Also notice the ways students check to see if their match is correct and how they explain and justify a match to each other. You can use this information to focus a plenary whole-class discussion.

**Support student reasoning.**

* Try not to make suggestions that move students towards a particular match. Instead, ask questions to help students to reason together.
* If you find one student has produced a solution for a particular match, challenge another student in the group to provide an explanation.
* If you find students have difficulty articulating their decisions, then use the sheet Suggested questions and prompts to support your own questioning of students.
* If the whole class is struggling on the same issue, you could write a couple of questions on the board and hold an interim, whole-class discussion. You could ask students who performed well in the assessment to help struggling students.
* During the small-group work, make a note of student approaches to the task, and to support student reasoning.

**Part 3: PLENARY (SUMMARY) WHOLE-CLASS DISCUSSION Time to Allot: (10 minutes)**

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| After the students have had completed the collaborative activity with their groups have them share their processes and completed graphs with the class.  Discussion prompts should be made up of your original guiding questions and notes about student approaches. Some other discussion prompts are listed below: |
| NOTE: *“Scribing” helps to increase student buy-in and participation. When a student answers your question, write the student’s name on the board and scribe his/her response quickly. You will find that students volunteer more often when they know you will scribe their responses – this practice will keep the discussions lively and active!* |

Encourage other students to challenge their explanations.

***Allow students time to collaborate as much as possible.***

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| **Part 4: Improving Solutions to the Assessment Task** | **Time to Allot: ( 20 minutes)** |
| The Shell MAP Centre advises handing students their original assessment tasks back to guide their responses to their new Post-Assessment (which is sometimes the exact same as the Pre-Assessment). In practice, some teachers find that students mindlessly transfer incorrect answers from their Pre- to their Post-Assessment, assuming that no “X” mark means that it must have been right. . Until students become accustomed to UNGRADED FORMATIVE assessments, they may naturally do this. Teachers often report success by handing students a list of the guiding questions to keep in mind while they improve their solutions.  *Look at your original responses and think about what you have learned this lesson.*  *Using what you have learned, try to improve your work.*  If you have not added questions to individual pieces of work then write your list of questions on the board.  Students should select from this list only the questions they think are appropriate to their own work.  They are to complete this post-assessment to the best of their ability without any prompts from the teacher. | |

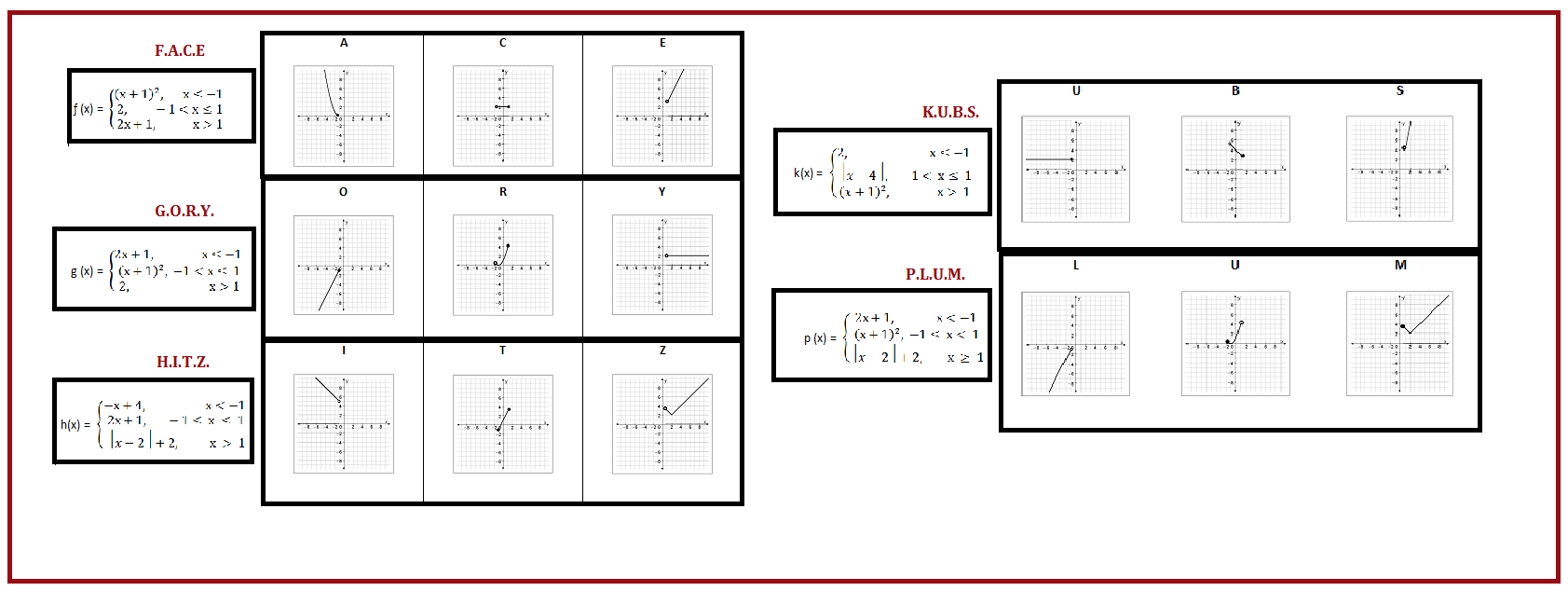
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| PRE-ASSESSMENT (Answer Key) |
| **ASSESSMENT TASK:** |
| Characteristics of Piecewise Functions |
| Characteristics of Piecewise-Defined Functions   1. Write the algebraic representation for the given piecewise function graph.     The answers here may vary. Some student may choose to write this piecewise-defined function with 3 linear functions. They may also have differences in their domain restrictions. As long as their domain restrictions do not over-lap they are correct.   1. Graph the piecewise-defined function: ƒ (x) = |

Collaborative Activity (Answer Key)

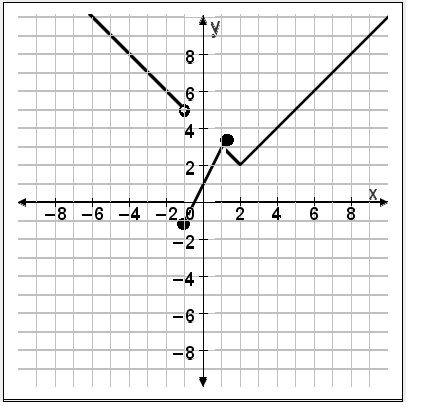
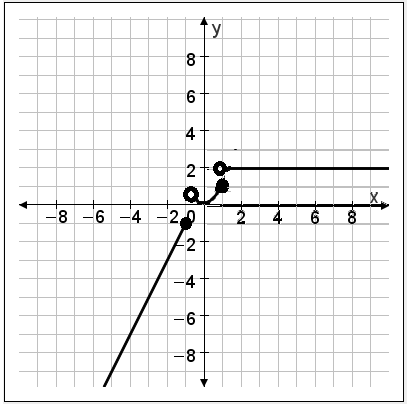
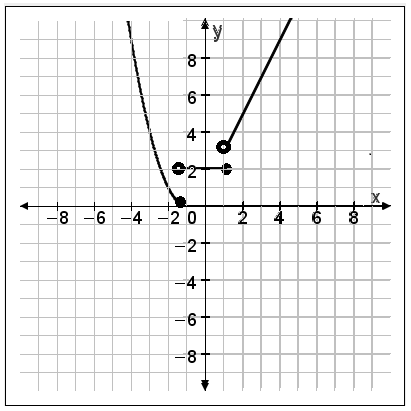
**CORRECT MATCHES FOR COLLABORATIVE ACTIVITY**

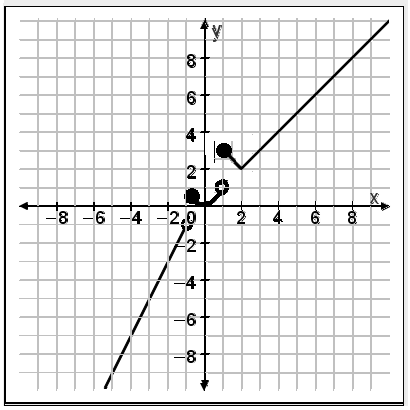
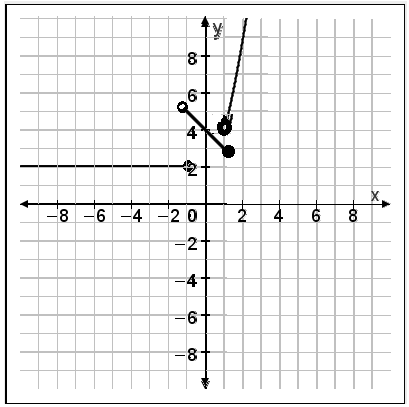
The functions were matches so that they spelled several four-letter words…for instance, f(x) matches with A, C, and E, in that order. So the teacher can quickly look at a student’s matches and know that if they see f, A, C, and E, that “F.A.C.E.” is in fact a correct matching of the three piecewise-defined function.

The key words to remember for quick verification of matches are FACE, GORY, HITZ, KUBS, and PLUM.



Completed Graphs (in order…FACE, GORY, HITZ, CUBS, and PLUM)

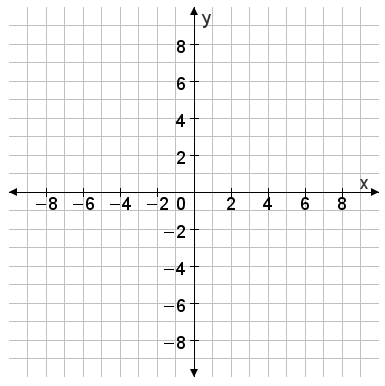
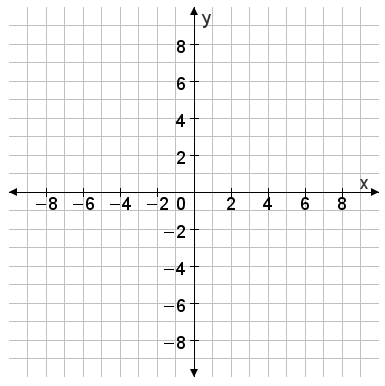


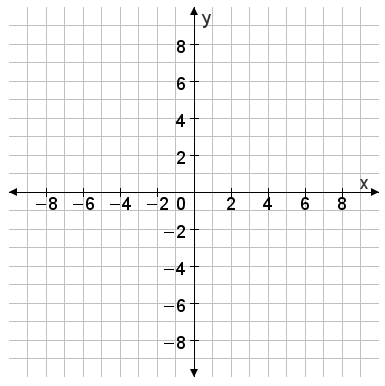
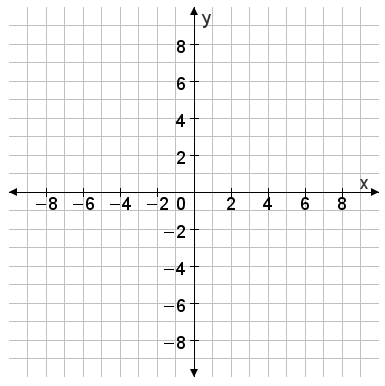


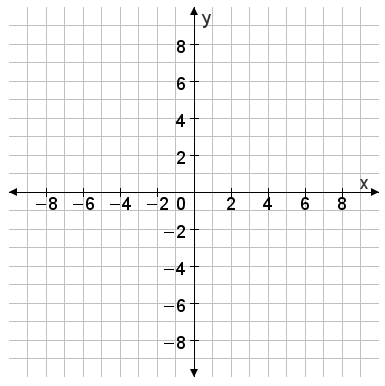
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| *Card Set A*  |  |  |  | | --- | --- | --- | | **Z** | **O** | **U** | | **C** | **I** | **E** | | **R** | **T** | **A** | |
| |  |  |  | | --- | --- | --- | | **Y** | **M** | **B** | | **S** | **U** | **L** |  |  |  | | --- | --- | | ƒ (x) = | g (x) = | | h(x) = | k(x) = | | p (x) = |  | |

# *Final Graphs*

(This sheet is provided if you would like students to graph entire piecewise functions on one page; using gridded chart paper as an alternative is suggested. This allows students to display their work).



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| **Lesson Day Warm-Up** |
| *In a pinball game, the ball rolls down a ramp, then is hit from beneath by a spring-loaded hammer, which tosses the ball up into the air onto a horizontal “JACKPOT” bar. From there, the ball rolls to the right, as sketched below.*     1. **How many different “parent function” shapes do you see in the path sketches?** 2. **What types of functions do you see (i.e. linear, quadratic, cubic, etc).** 3. **How could you find the equation for each of these paths?** |

**Collaborative Activity Instructions:**

* ***You have been paired into groups based upon your responses on the Pre-Assessment you were given.***
* ***Your goal is to match the graph of each piece of the piecewise-defined function with the card that has its algebraic representation.***
* ***After matching the three you believe belong together, draw the total graph on the grids provided for you.***
* ***Take turns matching pieces that you believe belong together. Each time you do match pieces, explain your reasoning clearly and carefully to your group.***
* ***It is important that you both understand your matches. If you do not agree or understand ask your partner to explain their reasoning. You both are responsible for each other’s learning.***

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| **Collaborative Plenary Discussion Questions:** |
| You are given a graph and an equation.   * How many “pieces” does each algebraic representation need? * What type of equation would be used to represent each piece of the graph? * What would the equation for those function families look like? * What does an open circle mean or represent in an inequality? * What does the closed circle represent? * Read the domain restrictions aloud and follow along by tracing with your finger on the graph. |